

TJUSAMO Practice 10 - Random Funness!

PDiao06

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1. Prove that if n is a positive integer such that $2n + 1$ and $3n + 1$ are both perfect squares, then n is divisible by 40.
2. (Ross) What are all integer polynomials $p(x)$ that only take on prime values when evaluated at every integer.
3. (USAMO 1974) $p(x)$ is a polynomial with integral coefficients. Show that there are no solutions to the equations $p(a) = b, p(b) = c, p(c) = a$, with a, b, c distinct integers.
4. (USAMO 1980) Find the maximum possible number of three term arithmetic progressions in a monotone sequence of n distinct reals.
5. (IMO 1977) Let a and b be positive integers. When $a^2 + b^2$ is divided by $a + b$, the quotient is q and the remainder is r . Find all ordered pairs (a, b) such that $q^2 + r = 1977$.

I guess by random fun I meant NUMBER THEORY!