

TJVMT POTW

Set #2

11/14/07 - 11/21/07

1P. Prove that the area of any triangle whose vertices have integral coordinates is half of an integer.

2S. The real number line between 0 and 1 is initially colored orange. It is first split into five equal segments, of which the second and fourth segments(not including their endpoints) are colored blue. Then each of the three orange segments left over is again split into five equal segments, of which the second and fourth segments(again, not including their endpoints) are colored blue. This is done to each of the remaining orange segments ad infinitum. What is the probability that a real number randomly and uniformly selected between 0 and 1 will eventually be colored blue by this process?

3S. Let O be the origin, and X, Y, Z be points on the positive x-axis, y-axis, and z-axis, respectively, such that $\angle XYZ = 72^\circ$ and $\angle XZY = 49^\circ$. Let T be the plane of $\triangle XYZ$. Point P is a point in T such that OP is perpendicular to T . Find $\angle XPZ$.