

# TJVMT POTW

Set #5

12/5/07 - 12/12/07

This week's problem set will be in standard relay format. This means that every problem except the first will have a variable  $T$  somewhere in the problem, which should be replaced by the answer from the last problem. Then, solve the problems as you would normally. Problems 2, 4, and 5 are worth 7 points each. (You need not submit the answers to 1 and 3.)

1S. Find the smallest positive integer that is divisible by every positive integer less than 3125.

2S. Find the number of positive divisors of  $T$  that are also perfect cubes.

3S.  $\triangle ABC$  is equilateral. Let  $D$  be a point outside  $\triangle ABC$  such that  $\angle ADC = 120^\circ$ . Given that  $AD = \frac{T}{5}$  and  $BD = \frac{T}{5} + 5$ , find the length of  $CD$ .

4S. There are  $T$  distinct pairs of socks in a sock drawer and  $T$  people who want a pair of socks. Each person randomly takes a left sock, then randomly takes a right sock. What is the probability that at least one person will be wearing a matching pair of socks?

5P. Let  $a_n$  be a sequence of real numbers. Let  $a_0 = T$  and let  $a_1$  be a positive integer. Given that for every positive integer  $i$ ,  $a_{i-1} = |a_i - a_{i+1}|$ , prove that there is no real number  $m$  such that for every positive integer  $i$ ,  $a_i < m$ .