

TJ USAMO Practice 2 - PHP

Varsity Math Team

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What are those pigeons and why are you stuffing them in holes?

0. Show that among any n people, there are two who know the same number of people. (If A knows B then B knows A.)
1. S is a set of eleven numbers are chosen from the set $1, 2, \dots, 99, 100$. Show that two disjoint, non-empty subsets of S have the same sum.
2. Determine the smallest integer n with the property that for any n distinct real numbers a_1, a_2, \dots, a_n , there exist integers i and j such that

$$0 < \frac{a_i - a_j}{1 + a_i a_j} < \sqrt{2} - 1$$

3. (Japan) Determine the smallest integer n with the property that in every n -digit number, there exists a string of consecutive digits whose product is a perfect square.
4. P is a convex region of area greater than 4 that is symmetric about the origin. Show that P contains a lattice point other than the origin.
5. (Poland) Suppose a triangle can be placed inside a square of unit area such that the center of the square is not inside the triangle. Show that a side of the triangle has length less than 1.