TJ USAMO Practice 1 - PROOFS!!

PDiao05

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You are all here to learn to write or at least practice writing proofs. Well, let me start by saying proofs are hard. Things you know from MathCounts or other short problem contests will be helpful, but there is so much to learn. In these practices we will try to teach you proving techniques, proof styles, and math facts that you need to know to prevail over the USAMO. Some notation that you may or may not be familiar with: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{Q}', \mathbb{R}$, and $\mathbb{C}$. They will denote the set of natural numbers, integers, rational numbers, irrational numbers, real numbers, and complex numbers respectively. We will go over more notation as it becomes relevant. The topic of this first practice is the actual writing of proofs.

For all of you who don't know about the test, if you do well enough on the AMCs and the AIMEs you can qualify to take the USAMO. This test is 4.5 hours for each of two days with 3 proofs per day. Each proof is worth up to seven points. The grading system is harsh and generally a good start missing one crucial step will only get you 1 or 2 points. A 6 is reserved for an almost perfect proof, and sevens are for perfect proofs. They don't like to give threes, fours, or fives. To make it into the USAMO you need a good enough index approximately weighted one point for each amc point and 10 points per AIME. The specifics of qualification are actually more vague, but you can ask us any questions you have about that.

The importance of proof writing style is huge. You should assume that the person who is grading your paper is both dumb, mean, stupid, and mean. In reality the people grading your papers are probably professors who don't know anything about contest math. You must state everything out to them explicitly. The deputy IMO team leader said basically that while grading the USAMO she ran around to check the other people's work. She actually changed about 20 zeros into sevens on the test. So when we do contests and we give you the style points, think of that as a probability. The probability you will get the points you deserve. So style is extremely important, it is worth taking the time to write a problem up nicely and neatly and correctly.

Here are some tips and advice from our more experienced members:

- "Read and know what the problem is," Jesse Geneson
- "Write legibly," Mark Hou
- "Don't write on the backs of your papers," Peter Diao

- “You don’t need to give how you thought of your proof just explain your proof clearly,” Peter Diao
- “Make a list of what you know and what you want to know,” Mark Hou
- “Try some easier examples to get a feel for what the answer should be,” Peter Diao
- “If you feel like you aren’t getting anywhere with one line of attack or feel like you are trying to force something to work, try something else,” Peter Diao
- “When quoting theoroms, make sure you show ALL the requirements are fulfilled and what exactly the theorom states,” Peter Diao
- “Don’t look at other people’s papers,” Lisa Marrone
- “Have fun, don’t be too serious, don’t stress,” Lisa Marrone
- “No.” Peter Diao
- “Don’t use Muirhead. Write out the AM-GM,” Daniel Schafer
- “Don’t submit scratchwork,” paraphrased from an incomprehensible sentence that Daniel Schafer uttered
- “Outline your cases in casework very clearly and explain why those are all of the cases,” Peter Diao
- “State what your base case and inductive hypothesis for inductions are and finish with a sentence such as ’the induction is complete’” Peter Diao
- “Tell us what your assumption is for proof by contradiction,” Peter Diao
- “For geometry, get yourself a good compass, ruler, and protractor so you can draw good diagrams,” Peter Diao
- “Don’t write in the margins of the paper or write in a spiral,” Daniel Schafer
- “Keep it real, except when trigonometry is involved,” Jesse Geneson
- “Write in pen if you are going to be faxing your solutions,” Daniel Schafer

1 Practice

Solve and write up these proofs.

1. Prove: $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$

2. Prove: $\sum_{i=1}^{n} i^2 = \frac{n * (n+1) * (2n+1)}{6}$

3. Find the number of digits in base 2 of the repeating part of 1/167.

4. In a complete K_6 graph colored with two colors, prove that there must be monochromatic triangle.