TJ USAMO Practe 5- Contest 1

VMT Officers

November 3, 2005

1. (JGeneson) Given positive reals x, y, z satisfying $2^{x^2}3^{y^3}5^{z^5} = 7$. What is the maximum value of xyz?

2. (2 USAMO 1986) Five Professors attended a lecture. Each fell asleep just twice. For each pair there was a moment when both were asleep. Show that there was a moment when three of them were asleep.

3. (JGeneson) Prove that the following is rational:

$$\sum_{k=1}^{\infty} \frac{k^{2005}}{2005^k}$$

4. **Bonus!** (WOOT) Given a circle with six distinct points on it, choose three of them and form a triangle. Find the centroid of this triangle. Take the remaining three points and form a triangle and finds its orthocenter. Form the line between these two points. You can do this in 20 different ways, prove that they all concur at one point. Mark and Haitao can't get credit for this.