TJUSAMO Problem Session #1

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March 5th, 2007

1. Find all sets of 500 consecutive naturals whose sum is a perfect square.

2. (IMO 2000 #1) AB is tangent to the circles $CAMN$ and $NMBD$ and parallel to CD. M lies between C and D on the line CD. The chords NA and CM intersect at P; the chords NB and MD intersect at Q. The rays CA and DB intersect at E. Prove that $PE = QE$.

3. Find all ordered quadruples of naturals (a, b, m, n) such that $a^m - b^m = 2^n$.

4. Let $a_1, a_2, \ldots, a_n$ be distinct integers. Show that the polynomial $P(x) = \prod_{i=1}^{n}(x - a_i) - 1$ is irreducible.

5. Let $a_1, a_2, \ldots, a_{111110}$ be integers. Prove that there exist 55555 distinct values $i_1, i_2, \ldots, i_{55555}$, such that $\sum_{n=1}^{55555} a_{i_n} \equiv 0 \pmod{55555}$.