TJUSAMO Contest #1

2 hours

November 29th, 2007

Each problem will be scored out of seven points similarly to the actual USAMO, with an additional style score based on how well your proof is written and how elegant your solution is. Your style score will be a decimal between 0 and 1 and will be multiplied with your math score for each problem to determine your actual score. Also note that for this contest, the problems are not necessarily ordered in increasing difficulty, though it has been publicly announced that the first problem is trivial. Good luck!

1. Given a positive integer n, prove that there exists an arithmetic sequence of length n that contains all distinct integers, but no perfect squares.

2. For any odd integer a that is greater than 5 and not divisible by 5:
a) Prove that $a^8 + 98a^4 + 1$ is divisible by 100.
b) Prove that $\frac{a^8+98a^4+1}{100}$ cannot be a prime integer.

3. Let P be a point inside $\triangle ABC$. Let D, E, F be points on the sides BC, CA, AB, respectively, such that no two points of the points labelled so far are the same. Given that P, F, A, E are concyclic(points that can be drawn on the same circle), and P, D, B, F are concyclic, prove that P, E, C, D must also be concyclic.