IMO Guts Round

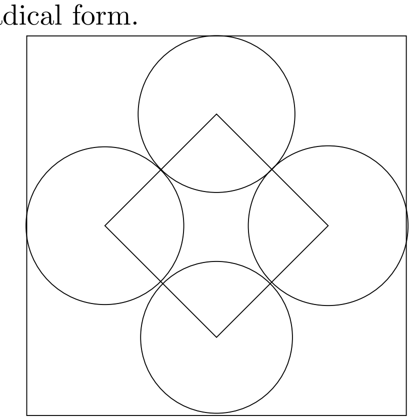
VMT Math Team

November 13, 2004

1. A man cannot decide of what he is going to wear. He has 5 different hats, 3 different pairs of shoes, 6 different shirts, and 4 different pairs of pants. He decides that he will try on every combination of clothing (exactly one of each item). If it takes him 30 seconds to try on an arrangement, how long will it take him to try every combination?

2. If for every chumbawumba there are 3 poppawoppas, 4 poppawoppas for every 5 meebledeebles, and 7 oogaboogas for every 6 meebledeebles, then what is the ratio of chumbawumbas to oogaboogas?

3. If the area of the small diamond is four, then what is the area of the large square? Express your answer in simplest radical form.



4. An expression is formed by aternatively adding and subtracting consecutive odd integers starting with 1 and ending with 1477 as indicated. What is its value?

$$1 - 3 + 5 - 7 + 9 - \cdots - 1475 + 1477$$

5. Menyoung and Eric are racing in the 1500 meter run. Menyoung keeps up a constant pace of 500 meters per 2 minutes, while Eric prefers to alternate between running 400 meters in 1 minute and walking 500 meters in 3 minutes. If they start running at the same time, who wins the race?

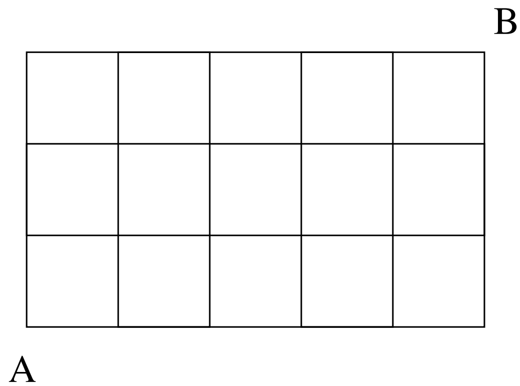
6. Several people are standing in a straight line. Bill is the 8th person from the front of the line. He is also the 12th person from the back of the line. How many people are in the line?

7. Maya had to paint an area of 10240 square units. She decided that each day she would paint half of the unpainted part of the previous day until she was about to leave less than one

square unit unpainted, in which case she would stay and finish the job that day. Using this strategy, how many days did it take Maya to finish painting?

8. $S(N)$ is the sum of the digits of N. What is the value of $(S(2004))^2 - S(2004^2)$?

9. How many paths of length 8 are there from A to B that travel only along the edges of the unit squares?



10. Suppose 300 wumpii are living in Wumpusville. If 172 wumpii can fly, 236 wumpii can swim, and 37 can do neither, how many wumpii can both swim and fly?

11. $\frac{1}{6}$ of the cars manufactuerd by a company are SUVs and the rest are Sedans. If $\frac{1}{11}$ of the cars produced are blue and $\frac{1}{3}$ of the SUVs are blue, then what fraction of the Sedans are blue?

12. What is the sum of all possible four digit positive integers which use the digits 2, 3, 5, and 6 each exactly once?

13. Starting at home, Brenda walked 1 block North, 2 blocks East, 3 blocks South, and 4 blocks West, and so on, each time turning 90 degrees to her right and walking one block further. After walking a total of 528 blocks, how many blocks is Brenda from her home? Express your answer in simplest radical form.

14. At Luigi's pizza place, Jimmy can order a small, medium, or large pizza with any combination of toppings. If he chooses exactly two of pepperoni, mushrooms, onions, or green peppers as his toppings, how many different types of pizza can Jimmy order?

15. What is the degree measure of the smaller angle formed by the minute and hour hands of a 12-hour clock at exactly 5:48?

16. Ricky drives to Carnegie Mellon University at an average speed of 55 miles per hour. Once there, he realizes that he has left his lap top at home. If he drives home along the same route this time averaging 66 miles per hour, then what is his average speed for the entire trip?

17. List all ordered pairs (x, y) of integers for which $xy = x + y$.

18. A sequence of numbers $a_1, a_2, a_3, a_4, \ldots$ is defined by $a_1 = 7$, $a_2 = -6$, and $a_n = a_{n-1} - a_{n-2}$. What is the sum of the first 1351 terms of this sequence?

19. The units digit 2 of a six-digit number is removed, leaving a 5 digit number. The removed units digit is then placed at the far left of the five-digit number, making a new six-digit number. If the new number is $\frac{1}{3}$ of the original number, what is the sum of the digits of the original number?

20. What is the maximum number of points of intersection between 3 lines and two circles?

21. In triangle ABC, $\angle C$ is right. D is the midpoint of $\overline{BC}$ and E is the midpoint of $\overline{AC}$. If the area of triangle CDE is 10, what is the area of $AEDB$?

22. $\overline{AC}$ is the base of isosceles triangle ABC. Points D and E are on $\overline{AC}$ and $\overline{AB}$ respectively so that $AD = DE = EB$. If $m\angle A = 20°$, then what is the degree measure of $\angle CBD$?

23. If x is a real number such that $x - \frac{1}{x} = 3$, then what is the value of $x^3 - \frac{1}{x^3}$?

24. List as many primes $p < 1500$ with the property that $p + 2, p + 6$, and $p + 8$ are also prime as you can find. You will recieve a half of a point for each correct answer.