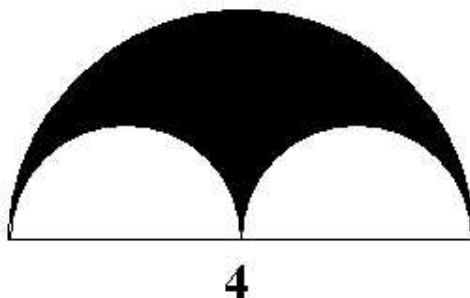


# IMO Individual Round

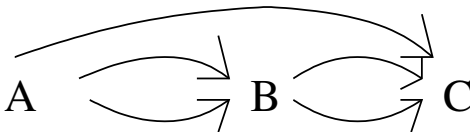
VMT Math Team

November 13, 2004

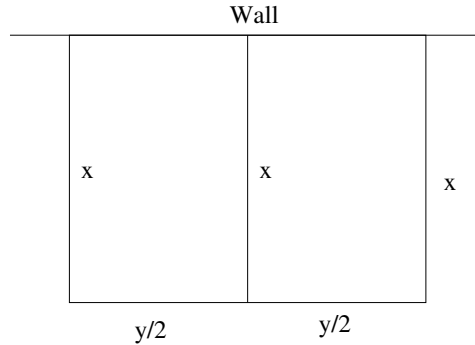
1. What is the product of all of the integers from -100 to 100 inclusive?
2. Find the area of the shaded region.



3. If  $x + y + z = 12$  and  $2x + 2y + z = 16$ , find  $z$ .
4.  $a$  is 50% of  $r$ , and  $b$  is 30% of  $r$ . What percent of  $r^2$  is  $ab$ ?
5. There are four people: Alice, Bob, Charlie, and Donatello. Alice has twenty cards, Bob has fifteen cards, Charlie has ten cards, and Donatello has twenty cards. If Alice gives one fourth of her cards to Bob, who then gives half of his cards to Charlie, who then gives a tenth of his cards to Donatello, how many cards did Bob lose?
6. How many paths are there from point A to point C? Count only paths which follow the directions of the arrows.



7. You have a rectangular fence up against a wall. The fence is  $x$  feet in length and  $y$  feet in width. The area of the enclosed region is 800 square feet. You split the area in half by adding another fence, after which you have used 100 feet of fence. If  $x > 15$ , what is the ordered pair  $(x, y)$ ?

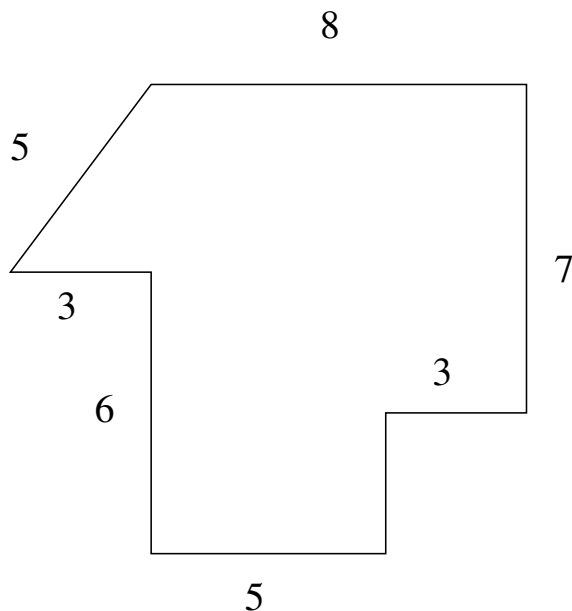


8. There are 52 cards in a folder: 13 of each of the four suits. You are given 4 of the clubs. If you are handed a randomly selected fifth card, what is the probability that it is also clubs?
9. What is the minimum value of  $x + \frac{9}{x}$  where  $x > 0$ ?
10. Let  $x \Delta y = \frac{x^2 - y^2}{2xy}$ . What is the value of

$$\frac{4(4 \Delta 2)}{6 \Delta 3}$$

11. Two people begin at the same point. They walk in opposite directions for three miles each. Then, each person turns to their right 90 degrees, and walks for four more miles. How many miles apart are the two people?
12. Today, Leopold is six years older than Megan. In nine years, Megan's age will be  $\frac{7}{9}$  of Leopold's age. How old is Megan now?
13. A woman buys a purse worth 150 dollars. She receives a 15% discount from the store, then pays a 4% sales tax. The woman pays in pennies. To the nearest penny, how many pennies did she pay?
14. Frederick was swimming up a river at a speed of 4 mph, then he turned around and swam downstream at a speed of 8 mph. He traveled the same distance both ways. What was his average speed in mph?
15. If the ratio of the circumference of a circle to the area of the circle is 1 : 2, then what is the radius of the circle?
16. Mark can run a lap every 50 seconds, Jonathan can run a lap every 60 seconds, and Jeff can run a lap every 70 seconds. If all three of them start running laps at the same time, how many seconds will elapse before they finish a lap at the same time?
17. The side of a cube is increased by 10%. By what percent does its volume increase?
18. If the ratio of  $a$  to  $b$  is 3 : 7 and the ratio of  $b$  to  $c$  is 3 : 2, what is the ratio of  $a$  to  $c$ ?
19. Billy runs across a track in 200 seconds. The next day he runs across in 198 seconds. He keeps doing this, day after day running 2 seconds faster each time until he runs across the track in 100 seconds. How much time does Billy spend running on the track?

20. Find the area of this shape (all angles are right angles except for the two in the upper left):



21. Find the greatest integer  $n$  less than 100 such that  $n$  has a remainder of 4 when divided by 5 and a remainder of 3 when divided by 4.
22. Geoff has \$40 in \$2 and \$5 bills. If Geoff has a total of  $x$  bills, compute the number of possible values of  $x$ .
23.  $A$ ,  $B$ ,  $C$ , and  $D$  are points on a line.  $D$  is the midpoint of  $BC$ . If  $AB = 10$ ,  $AC = 2$ , and  $BC = 12$ , then what is the length of  $AD$ ?
24. On a 50 question test, you get 2 points for each correct answer, and -1 point for each wrong answer. (Omitted questions neither lose nor receive any points.) Peter scored a total of 78 points and omitted 2 questions. How many correct answers did Peter get?
25. A photo has its width increased by 10% and its height decreased by 10%. What is the ratio in terms of area of the new photo to the original photo?
26. A state had three letters followed by three digits for the format of licence plates. In order to increase the number of plates available, the state change changed the format to five letters followed by one digit. What is the positive difference between the number of plates available with the new format and the number of plates available with the old format?
27. Mrs. Read can knit one pair of children's mittens with a ball of yarn 5 inches in diameter. How many pairs of mittens can she knit with a ball of yarn 30 inches in diameter?
28. Simplify fully:  $(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \cdots (1 - \frac{1}{2004})$
29. The number 6 is a perfect number because it is the sum of all of its positive divisors except itself. ( $6 = 1 + 2 + 3$ ). What is the next smallest perfect number?

30. The sum of two numbers is 60. The sum of their reciprocals is  $\frac{1}{12}$ . What is the product of the two numbers?
31. Compute the sum  $1 + 3 + 5 + 7 + \cdots + 2005$ .
32. Arrange in increasing order:  $2^{35}, 3^{21}, 5^{14}, 30^7$ .
33. In triangle  $ABC$ ,  $m\angle A = 15^\circ$ . The perpendicular bisector of  $\overline{AB}$  intersects  $\overline{AC}$  at  $D$ . The angle bisector of  $\angle CBD$  intersects  $\overline{CD}$  at  $E$ . If  $BE = CE$ , then what is the degree measure of  $\angle BED$ ?
34. In a non-constant arithmetic sequence that begins with 8, the first, third, and sixth terms (in that order) form a geometric progression. What is the 10th term of the arithmetic sequence?
35. 118,785 is divisible by just 8 positive integers. What is its largest prime factor?