

IMO Team Round

VMT Math Team

November 13, 2004

1. When positive integers x and y are divided by 13, the remainders are 7 and 2, respectively. What is the remainder when x^2y^2 is divided by 13?
2. Two standard dice are rolled. What is the probability that the product of the two numbers rolled exceeds 10? Express your answer as a common fraction.
3. The mean, median, and mode of the set $\{72, 55, 63, x, 30\}$ are all equal. Find x .
4. 1111 in base x is equal to 40 in base ten. What is x ?
5. Thomas Jefferson is hosting a chess tournament with 150 entrants. Assume that every chess match results in 1 winner and 1 loser (no ties are allowed.) Each round, the remaining competitors pair up and play a game with only the winners moving up to the next round. If there are an odd number of entrants in a certain round, 1 is randomly selected to automatically move to the next round and does not play in a match for the round. How many chess matches are required to determine the winner of the tournament?
6. What is the largest value of n for which $100!$ is divisible by 10^n ?
7. In the equation $ab = 1254$, a and b are both two digit numbers. What is the greatest possible value of $a + b$?
8. Define the binary operator ∇ as $x\nabla y = xy - x - y + 1$. What is

$$\frac{1\nabla 49 + 3\nabla 7 + \frac{(-3)\nabla 2}{2}}{(4\nabla 2)\nabla 9}$$

in reduced form?

9. What is the number of positive integers that divide into 840 with no remainder?

10. In the figure below, $AB + BC + CD + DE = 16$ and $AB = DE + 2$. O is the center of the quarter-circle that has endpoints A and E , and C is on the quarter circle. Compute the area of the shaded region given that $m\angle ABC = m\angle CDE = 90^\circ$.

